Potential and timescales for oxygen depletion in coastal upwelling systems:

A box-model analysis

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Abstract

A simple box model is used to examine oxygen depletion in an idealized ocean-margin upwelling system. Near-bottom oxygen depletion is controlled by a competition between flushing with oxygenated offshore source waters and respiration of particulate organic matter produced near the surface and retained near the bottom. Upwelling-supplied nutrients are consumed in the surface box, and some surface particles sink to the bottom where they respire, consuming oxygen. Steady states characterize the potential for hypoxic near-bottom oxygen depletion; this potential is greatest for faster sinking rates, and largely independent of production timescales except in that faster production allows faster sinking. Timescales for oxygen depletion depend on upwelling and productivity differently, however, as oxygen depletion can only be reached in meaningfully short times when productivity is rapid. Hypoxia thus requires fast production, to capture upwelled nutrients, and fast sinking, to deliver the respiration potential to model bottom waters. Combining timescales allows generalizations about tendencies toward hypoxia. If timescales of sinking are comparable to or smaller than the sum of those for respiration and flushing, the steady state will generally be hypoxic, and results indicate optimal timescales and conditions exist to generate hypoxia. For example, the timescale for approach to hypoxia lengthens with stronger upwelling, since surface particle and nutrient are shunted off-shelf, in turn reducing subsurface respiration and oxygen depletion. This suggests that if upwelling winds intensify with climate change the increased forcing could offer mitigation of coastal hypoxia, even as the oxygen levels in upwelled source waters decline.
1. Introduction

Oxygen depletion leading to hypoxia and anoxia in coastal waters can negatively affect fisheries and local ecosystem functioning, particularly in benthic environments that support commercially important populations of groundfish and crustaceans (Bailey et al. 1985; Chan et al. 2008; Keller et al. 2015). Bottom waters over the shelf can become hypoxic, with variability over a variety of temporal and spatial scales (Bailey et al. 1985; Bailey 1991; Grantham et al., 2004; Chan et al., 2008; Peterson et al. 2013; Adams 2014; Siedlecki et al. 2015). In extreme cases low oxygen events lead to large areas of mortality popularly referred to as "dead zones." Such events have been reported with increasing frequency (Grantham et al. 2004; Chan et al. 2008) and associated with detrimental effects (Keller et al. 2015) in the Pacific Northwest, stimulating interest into processes controlling hypoxia (Hales et al. 2006; Connolly et al. 2010; Bianucci et al., 2012; Adams 2014; Peterson et al. 2013; Siedlecki et al. 2015). These studies indicate that both local respiration of organic matter and variations in the oxygen content of upwelled waters play a role in the seasonal development of hypoxia on shelves in the Pacific Northwest, while farther to the south along the California coast, the transport of low oxygen water masses onto the shelf and stratification are thought to be potentially more important drivers (Bograd et al. 2008; Send and Nam, 2012). Controls over interannual variability in the severity of hypoxic events, and specifically how hypoxia in upwelling systems is related to regional and basin scale processes, still remain unclear (Hales et al. 2006; McClatchie et al. 2010; Deutsch et al. 2011; Nam et al., 2011).

In coastal upwelling systems, equatorward winds drive surface offshore Ekman flow, drawing nutrient-rich water from the ocean interior to the surface and supporting
productive coastal ecosystems (Ryther 1969; Chavez and Toggweiler, 1995; Carr and Kearns, 2004). The close proximity of the seafloor to the productive coastal surface waters allows retention of oxygen-depletion potential in the coastal system, with the retention fraction determined by conversion of upwelled nutrient to sinking organic matter that is subsequently respired within shelf waters rather than transported offshore. The source waters for this upwelling originate from the low-oxygen ocean interior and so are subject to the mechanisms for interior oxygen depletion described above. Regional variations in wind forcing and other conditions also contribute to the variability of oxygen levels within these systems. For example, in the California Current System, the shelves are wider, upwelling weaker, and freshwater sources more prevalent in the north than in the south, all of which can affect local levels of hypoxia.

In general, the extent and rate of development of hypoxia in coastal upwelling systems depends on the timescales of competing physical and biogeochemical processes that determine the flushing of these waters and the retention within them of organic matter that represents an effective potential source of respiratory oxygen consumption. The role of source water oxygen level is relatively direct and has been highlighted as a potential driver of hypoxia in upwelling regions (Grantham et al., 2004; Bograd et al. 2008; McClatchie et al. 2010; Peterson et al. 2013). The factors that determine the rate and extent of on-shelf oxygen depletion are less understood, although the interplay between respiration and flushing is clearly important (Adams et al. 2013; Siedlecki et al., 2015).

Coupled numerical models have begun to give useful insights into the local dynamics of hypoxia in specific regions and systems (e.g., Peña et al., 2010; Siedlecki et
al., 2015), but we take a different approach here. We use instead a highly simplified box-
type model to explore systematically the potential for oxygen depletion in a broad class
of upwelling systems, and to suggest general relationships between the timescales of
physical flushing and the biogeochemical processes of production, respiration, and
sinking that determine this potential. Our goal is not to provide a specific prediction for
hypoxic behavior in a particular region at a particular time, but instead to develop a broad
quantitative characterization of the relative roles of the fundamental rates and parameters
in controlling the susceptibility of upwelling systems to hypoxia. We hope thereby also
to enable a general assessment of the likelihood of hypoxic behavior in any given system
for which a representative set of rates and parameters can be constructed or estimated.

With this goal in mind, we construct a highly simplified box model of the
upwelling system, and then subject it to a thorough analysis in model-parameter space,
the broadest summary aspects of which necessarily involve a relatively abstract
framework. The main results are given in Sections 3-5. Section 3 presents the most
concrete and accessible description, in terms of two specific examples. Section 4
progresses to a discussion of two abstracted elements, steady-state depletion and an
exponential timescale for evolution toward hypoxia, and their dependence on process
rates and parameters. Section 5 collects the most general results, framed in terms of
relative – rather than absolute – process rates, and broadly describing the susceptibility of
the model system to hypoxia through derived criteria on these relative rates.
2. Model description

2.1 Overview

The model represents integral balances for a cross-shelf slice of an alongshore-uniform continental shelf domain under upwelling conditions (Fig. 1). Motivated by observations in upwelling systems of elevated particulate organic carbon over the shelf in both the surface and bottom boundary layers (e.g. Karp-Boss et al., 2004; Hales et al. 2006; Holser et al., 2010) separated by a mid-water layer with low particle abundance, we partition the mid-shelf by depth into three control volumes, or boxes. The top box, with thickness $H$, represents a surface boundary layer, where both offshore physical transport and particulate organic matter production are confined. The bottom box, taken to have the same thickness $H$, represents the bottom boundary layer, in which onshore upwelling transport is confined. The interior water column between the surface and bottom boundary layers forms a middle box, with thickness $H_{mid}$, so that the ratio between the middle and boundary layer boxes is $\delta_H = H/H_{mid}$. A single nominal cross-shelf width $L$ is specified for all three boxes, and all modeled balances and quantities are computed per unit alongshore distance.

The model state variables are: $\Delta O_{bot} = O_s - O_{bot}$, the depletion of bottom-box oxygen concentration $O_{bot}$ from its fixed offshore value $O_s$; $C_{bot}$ and $N_{bot}$, the bottom-box particle and generic nutrient concentrations, respectively; and $C_{top}$ and $N_{top}$, the corresponding top-box concentrations. The model equations are

\[
\frac{d\Delta O_{bot}}{dt} = \frac{v}{\gamma'} r C_{bot} - \lambda \Delta O_{bot}. \tag{2.1}
\]

\[
\frac{dC_{bot}}{dt} = E_{bot} - r C_{bot}, \tag{2.2}
\]
\[
\frac{dN_{bot}}{dt} = \frac{r}{\gamma} C_{bot} + \lambda (N_s - N_{bot}), \quad (2.3)
\]

\[
\frac{dC_{top}}{dt} = \left( p \frac{N_{top}}{K + N_{top}} - r - s - \lambda \right) C_{top}, \text{ and} \quad (2.4)
\]

\[
\frac{dN_{top}}{dt} = \frac{1}{\gamma} \left( r - p \frac{N_{top}}{K + N_{top}} \right) C_{top} + \lambda (N_{bot} - N_{top}), \quad (2.5)
\]

where the sinking flux into the bottom box, \( E_{bot} \) in (2.2), is given in terms of \( C_{top} \) by

\[
E_{bot}(t) = \delta s^2 \int_0^t e^{-(r+\delta)(t-t')} C_{top}(t') d t'. \quad (2.6)
\]

The middle box is linked to the top and bottom boxes by particle sinking fluxes but is taken otherwise to be isolated, and the middle-box particle balance can thus be entirely represented by the integral transform (2.6) [see Appendix, Eq. (A.1)]. The derivation of (2.1)-(2.6) and the definitions of the parameters in these equations are summarized in Section 2.2. Parameter values are discussed in Section 2.3.

### 2.2 Formulation and parameter definition.

The physical circulation comprises wind-driven cross-shore Ekman transport \( U_E \) (e.g., Gill, 1982) per unit alongshore distance, analogous to an upwelling index (Schwing et al. 1996). Here \( U_E < 0 \) corresponds to equatorward wind forcing and upwelling conditions, with offshore transport out of the top box and compensating onshore transport into the bottom box and upwelling from the bottom box to the top box. In all cases \( U_E \leq 0 \), so that downwelling never occurs. Normalizing the transport by cross-shelf box area \( LH \) yields an upwelling rate \( \lambda \) (d^{-1}):

\[
\lambda = -U_E / LH \quad (2.7)
\]
Since all offshore transport from the top box is compensated by onshore flow to the bottom box, and top and bottom boxes are prescribed to have the same volume, $\lambda$ is also the flushing rate of the bottom box. For simplicity, downwelling and mixing between the boxes are not considered, consistent with the focus on advectively driven upwelling systems.

The dissolved O$_2$ concentration in the bottom box is denoted $O_{bot}$ (mmol m$^{-3}$), and its rate of change is determined by the respiration of bottom-box particles, with particle concentration $C_{bot}$ (mmol m$^{-3}$), and the divergence of onshore and upward upwelling-driven advective oxygen fluxes:

$$\frac{dO_{bot}}{dt} = \lambda (O_s - O_{bot}) - \frac{\nu}{\gamma} r C_{bot}. \quad (2.8)$$

In (2.8), the constants $r$ (d$^{-1}$) and $O_s$ (mmol m$^{-3}$) are, respectively, the carbon-specific particle respiration rate and the oxygen concentration of the deep offshore source waters drawn into the bottom box by the upwelling circulation. The ratio $\nu/\gamma$ represents the stoichiometric proportion of O$_2$ consumed per unit organic carbon during respiration, represented in terms of the ratios $\nu$, of O$_2$ consumed to nitrogen released during respiration, and $\gamma$, of organic carbon to nitrogen in average marine organic matter, with nominal, fixed values $\nu=10$ and $\gamma=7$, so that $\nu/\gamma = 10/7 \approx 1.4$ in (2.8). Equation (2.8) can be rewritten in terms of the oxygen depletion from the constant offshore value, $\Delta O_{bot} = O_s - O_{bot}$, yielding (2.1). This relative depletion is independent of $O_s$, and emphasizes on-shelf processes. Since downwelling and mixing between the boxes are not allowed, oxygen enters the bottom box only from offshore and the oxygen mass balances for the other boxes can be ignored.
Determination of $C_{bot}$ (mmol m$^{-3}$) is necessary for solution of (2.8) or (2.1), and this in turn requires linkage to the particle production in the surface box and its export to the bottom box by sinking. The balance (2.2) between respiration and the incoming sinking flux $E_{bot}$ (mmol m$^{-3}$ d$^{-1}$) controls the bottom-box particle concentration. Essentially all particles incorporated in surficial sediments are respired on short timescales very near the sediment-water interface, with respiration products rapidly returned to overlying waters (see Siedlecki et al., 2015; Bianucci et al., 2012), so no explicit term is included representing loss of $C_{bot}$ through long-term sediment burial, and sediment respiration is assumed to be incorporated in the bottom-box respiration term.

In the top box, nutrients are taken up and transformed directly into particles, and intermediate biological processes are only implicitly represented. As described by (2.4), $C_{top}$ production thus proceeds via a particle-specific rate constant, $P$ (d$^{-1}$), scaled by a Monod-form dependence on the concentration of a generic nutrient $N_{top}$ (mmol m$^{-3}$), with half-saturation constant $K$ (mmol m$^{-3}$). $C_{top}$ is lost in proportion to its concentration by respiration at the rate $r$, by export through vertical sinking at the specific rate $s$ (d$^{-1}$), and by transport offshore at the upwelling rate $\lambda$ (d$^{-1}$). Particles are assumed to sink faster than the upwelling velocity, so that there is no upwelling flux of particles. The specific sinking rate $s = S/H$, where $S$ (m d$^{-1}$) is the sinking speed.

The two boxes are linked by the dependence of the incoming particle flux $E_{bot}$ (2.6) on the sinking export flux $sC_{top}$. These fluxes would be instantaneously equal if the middle box were absent; however, the sinking flux is attenuated by respiration and temporally delayed during transit through the middle box. In (2.6), $\delta$ is the ratio of middle-box and top-box specific sinking rates,
\[
\delta = s' / s
\]

(2.9)

where \( s' = S' / H_{\text{mid}} \) and \( S' \) is the middle-box sinking speed, so that \( \delta = \delta_H \) when \( S' = S \).

The incoming sinking flux \( E_{\text{bot}} \) will differ most from the surface sinking flux for rapid respiration (large \( r \)) and long settling transit time (small \( \delta \)), which give large attenuation and delay (Appendix).

While surface nutrient levels are often high under upwelling conditions, they can fall to levels near \( K \) that limit production, and must be modeled explicitly. The top-box (\( N_{\text{top}} \)) and bottom-box (\( N_{\text{bot}} \)) nutrient balances, (2.5) and (2.3), include the same particle growth and respiration terms seen in (2.4) and (2.2), with opposite sign and scaled by \( \gamma \).

Nutrient is supplied to the top box by upwelling from the bottom box and transported offshore with the flushing rate \( \lambda \); it is supplied to the bottom box from offshore source waters with concentration \( N_s \) and upwelled with the same timescale.

2.3 Parameter values

The model as formulated depends only on the rate constants \( \lambda, p, s, \) and \( r \), the dimensionless specific-sinking-rate ratio \( \delta \), the half-saturation constant \( K \), and the initial (to be denoted by subscript 0, e.g., \( C_{\text{top0}} \)) and source-water (subscript \( s \), e.g., \( O_s \)) values of the modeled concentrations. The initial and source-water concentrations were fixed at nominal values (Table 1), based on previous observations of Oregon coast source and on-shelf water nitrate (Hales et al., 2005) oxygen (Hales et al., 2006) and particulate organic carbon (Karp-Boss et al., 2004; Hales et al., 2006; Holser et al., 2010). The dependence on \( K \) was examined for a wide range of values, \( 0 < K < 2 \text{ mmol m}^{-3} \). In addition, two specific levels of bottom oxygen depletion were used as indicators for hypoxia, 40 and 100 mmol m\(^{-3} \), which for a nominal source-water oxygen concentration of 100 mmol m\(^{-3} \),
correspond respectively to moderate hypoxia (60 mmol m\(^{-3}\) O\(_2\)) and severe hypoxia or anoxia (0 mmol m\(^{-3}\) O\(_2\)). Model geometry was fixed, with \(H = 20\) m and total depth 100 m, so that \(\delta = 1/3\) when \(S' = S\) (Table 2), but the general dependence on \(\delta\) was also examined. The rate constants must be positive and are further constrained by observational or laboratory estimates. The respiration rate \(r\) was fixed at 0.2 d\(^{-1}\), consistent with observations of particle-specific respiration rates for the Oregon coastal upwelling system (Wetz et al., 2008; Hales, unpubl. res.; F. Chan., pers. comm.).

Other parameters were allowed to vary to reflect ranges of possible upwelling and biogeochemical rates. We considered upwelling rates \(\lambda\) from 0.02 to 0.4 d\(^{-1}\), spanning a range of conditions from weak upwelling over a broad shelf (such as the Galician upwelling system; e.g., Herrera et al., 2008) to strong upwelling over a narrow shelf (such as the central-northern California system; e.g., Chavez and Messi, 2009). For example, upwelling wind stress 0.1 N m\(^{-3}\) and box-width \(L = 20\) km gives \(U_E \approx 0.5\) m\(^2\) s\(^{-1}\) and \(\lambda \approx 0.1\) d\(^{-1}\). In all cases, excepting a brief discussion of preliminary results for variable forcing, the upwelling rate \(\lambda\) was taken to be constant in time. We considered ranges of \(p\) from near 0 up to 2 d\(^{-1}\), capturing the fastest reported doubling times for fast-growing diatoms (Barber and Chavez, 1991; Lindley et al., 1995; Dugdale and Wilkerson, 1998; Chai et al., 2002). We considered sinking rates \(s\) ranging from near 0 up to 1 d\(^{-1}\), which, for the fixed 20-m thickness of the surface box, corresponds to a maximum physical sinking velocity of 20 m d\(^{-1}\), consistent with observed sinking velocities for large individual diatoms (Jamart et al., 1977; McDonnell and Buessler, 2010; 2012; Iverson and Ploug, 2010; 2013). Part of the analysis is based on relative rates, further generalizing these nominal parameter ranges.
To contrast the magnitudes and timescales of bottom oxygen depletion for systems dominated by rapidly growing and sinking particles and by slowly growing and sinking particles, two specific sets of biogeochemical rates were chosen as example cases (Table 2), denoted FP (‘Fast Particles’) and SP (‘Slow Particles’). The FP case represents a system dominated by rapidly growing and sinking plankton with a higher nutrient half-saturation constant, such as coastal diatoms, which are observed to dominate the biomass of the initial production response to coastal upwelling (e.g. Dugdale et al., 1998). The SP case represents a system with a slower biological response to upwelled nutrients and much slower particle sinking velocities, but a smaller half-saturation constant and thus less sensitivity to nutrient level.

3. Results: FP and SP examples

3.1 Temporal evolution

In this section, the temporal evolution of the selected-parameter FP and SP cases (Sec. 2.3, Table 2) is considered for a simple seasonal upwelling scenario, in which the system is initialized with low surface particle and nutrient concentrations and then forced with constant upwelling-favorable winds, $\lambda = 0.1 \text{ d}^{-1}$ for 150 days, a period comparable to the maximal extent of a mid-latitude eastern boundary upwelling season. These examples illustrate the response of the bottom-box oxygen depletion ($\Delta O_{\text{bot}}$; Eq. 2.1) to the bottom-box particle dynamics, and the relation of these to the evolution of the surface particle dynamics, and, in turn, the coupling of both to the upwelling circulation.

After 10 days of sustained upwelling, $\Delta O_{\text{bot}}$ even in the FP case has reached only $\sim 2 \text{ mmol m}^{-3}$ (Fig. 2a), but, after 20 days, it reaches nearly $30 \text{ mmol m}^{-3}$. The moderate hypoxia threshold ($\Delta O_{\text{bot}} = 40 \text{ mmol m}^{-3}$), is surpassed by day 22, and the extreme
hypoxia threshold ($\Delta O_{bot} = 100 \text{ mmol m}^{-3}$) by day 31. The final $\Delta O_{bot}$ for this scenario is over 200 mmol m$^{-3}$, more than enough to deplete all of the oxygen from a broad spectrum of ocean interior waters. This rapid and intense depletion suggests that the near-bottom waters of a simple upwelling system dominated by fast growing and sinking particles and subject to a constant upwelling forcing would spend the majority of the upwelling season in an anoxic state.

Oxygen depletion is driven by the respiration of accumulating organic particles in the bottom layer ($C_{bot}$). To understand the mechanism behind this behavior, it is useful to examine more closely the evolution of the particle concentrations. The rapid rise in $\Delta O_{bot}$ for FP is a delayed response to the rise in $C_{bot}$, which builds from $\sim 3 \text{ mmol m}^{-3}$ at day 10 to $\sim 30 \text{ mmol m}^{-3}$ at day 20, eventually approaching a value of $\sim 70 \text{ mmol m}^{-3}$ (Fig. 2). The rise in $C_{bot}$ is driven, ultimately, by the sinking of particles produced in the surface box, $C_{top}$, into the bottom box. The $C_{top}$ response is characterized by a rapid initial rise in concentration, with $C_{top}$ at day 10 greater than 9 mmol m$^{-3}$, nearly an order of magnitude higher than the initial condition (Fig. 2b). By day 20, $C_{top}$ is near the peak value of $\sim 70$ mmol m$^{-3}$; however, unlike the deep box dynamics, the particle evolution advances through a local minimum just after day 25, followed by a slower secondary rise towards the steady-state value of $\sim 60$ mmol m$^{-3}$. The details of the fluctuations during this transition from the initial growth phase toward the later asymptotically steady regime are dependent on the initial concentration values as well as the other model parameters.

The rise in $C_{top}$ is initially sustained by the upwelling of $N_{bot}$, but in all cases, consumption eventually exceeds the upwelled source, and $N_{top}$ begins to decline. As long as $N_{top}$ remains much greater than $K$, so that $N_{top}/(K + N_{top}) \approx 1$, $C_{top}$ growth is
approximately exponential at the maximal (i.e., nutrient-saturated) accumulation rate $\alpha$,

$$\alpha = p - r - s - \lambda.$$  (3.1)

Even as $C_{\text{top}}$ is growing exponentially, $N_{\text{top}}$ also initially grows, due to upwelling of the high $N_{\text{bot}}$ waters. By day 10, for FP, $N_{\text{top}}$ has risen to near its peak value of over 20 mmol m$^{-3}$. At that point $C_{\text{top}}$ has reached such a high level that the consumption of $N_{\text{top}}$ begins to exceed the upwelled supply, and $N_{\text{top}}$ begins to fall, reaching a minimum of just over 1 mmol m$^{-3}$ shortly after day 20. $N_{\text{top}}$ then rebounds, passing through a weak local maximum of about 3 mmol m$^{-3}$ near day 25, corresponding to the $C_{\text{top}}$ local minimum, before settling toward the asymptotic value just under 3 mmol m$^{-3}$. The post-minimum fluctuations are driven by the dynamics of the supply of $N_{\text{bot}}$ to the surface (Figure 2b).

Following the initially slow rise of $\Delta O_{\text{bot}}$, $N_{\text{bot}}$ has risen by only 0.2 mmol m$^{-3}$ by day 10, and $< 3$ mmol m$^{-3}$ by day 20, the time of the $N_{\text{top}}$ minimum. After that, $N_{\text{bot}}$ follows a slow rise towards an asymptotic value of $\sim 55$ mmol m$^{-3}$. This interplay between the surface nutrient concentration evolution and the increasing contribution of bottom nutrients supplied by respiring $C_{\text{bot}}$, plays a large role in the evolution of oxygen depletion of the bottom shelf water.

Oxygen depletion for the SP case is strikingly distinct from the FP case. At 10- and 20-day benchmarks, $\Delta O_{\text{bot}}$ (Fig. 3a; light line, Fig. 2a) is $< 0.02$ and $< 0.08$ mmol m$^{-3}$, orders of magnitude less than seen for the FP-case. Oxygen depletion for SP never even approaches the moderate hypoxia threshold, instead asymptotically approaching only $\sim 9$ mmol m$^{-3}$. Similar to the FP case, the SP $\Delta O_{\text{bot}}$ evolution is a fairly straightforward
connection to the $C_{bot}$ evolution: a rise that slightly lags the $C_{bot}$ rise, with each case reaching asymptotic values with $\Delta O_{bot} \sim 3C_{bot}$.

Like the FP case, SP $\Delta O_{bot}$ dynamics are ultimately tied to production of $C_{top}$ and sinking through the mid-water column into the bottom box, but similarity ends there. Growth of $C_{top}$ (Figure 3b) is, expectedly, slower for the slower specific growth rate $p$, with $C_{top}$ at the 10 and 20-day benchmarks of only $< 3$ and $< 7$ mmol m$^{-3}$, 3 and 10-fold lower than FP at the corresponding times. SP $C_{top}$ reaches its local maximum over 35 days later, and then drops and proceeds to an asymptotic condition without passing through a local minimum. The SP $C_{top}$ dynamics are controlled similarly to the FP case: low initial $C_{top}$ allows $N_{top}$ to grow in response to upwelling until exponential $C_{top}$ growth leads to $N_{top}$ consumption exceeding upwelling supply, causing nutrients to fall to a low and nearly constant value approaching the limiting $K$ value. However, the maximum and asymptotic $C_{top}$ values for SP, $\sim 185$ and $\sim 163$ mmol m$^{-3}$, respectively, are each nearly 3x higher than corresponding FP values.

3.2 Oxygen depletion and retention efficiency

Oxygen depletion for a given flushing or upwelling rate is determined by the overall amount of respiration of $C_{bot}$; this is ultimately linked to the production of $C_{top}$ and thus the upwelling-rate dependent delivery of nutrients to the surface. This suggests that upwelling will favor hypoxia when the surface particle dynamics and export are fast enough to keep up with upwelling transports and focus that respiratory potential on the bottom box (Bakun 1990; Bakun et al., 2015). The latter capacity can be quantitatively characterized by the retention efficiency, $c$, defined here as:
where the numerator represents the integrated consumption of oxygen by respiration of particles in the bottom box, and the denominator measures the integrated potential for respiration from nutrient supplied to the system by upwelling, i.e., the respiration that would result from its complete transformation into respiring bottom-box particles. If rapid particle growth and sinking efficiently converts upwelled nutrients into $C_{bot}$, the retention efficiency $c$ will be large and the system will rapidly lose oxygen. If slow particle growth allows potential respiration to escape offshore in the form of un-utilized nutrient, retention efficiency and oxygen depletion will be low. Efficiency and depletion will also be low if slow sinking allows $C_{top}$ to be transported offshore, or prevents surface export from being effectively transferred to $E_{bot}$.

Temporal evolution of the retention efficiency (3.2) for the FP and SP examples demonstrates its fundamental relation to $\Delta O_{bot}$ evolution (Fig. 4a). The FP case shows a rapid rise in retention of the respiration potential of upwelled source water nutrients towards an asymptotic value approaching 0.5; in contrast, the SP retention never exceeds 0.02 (Fig. 4a). The differences in retention in these two cases are primarily due to differences in loss of respiration potential by offshore transport. In the FP case, about half of the respiration potential has been exported offshore from the surface box, in a roughly 2:1 ratio between export of $C_{top}$ that avoids sinking and $N_{top}$ that has not been incorporated in net $C_{top}$ production. In the
SP case, nearly 75% of the respiration potential is lost via offshore export of $C_{\text{top}}$ and $N_{\text{top}}$.

If there is a steady-state balance between respiration and flushing in (2.1), such as is approached for large times in the FP and SP cases, then by (2.10) the retention efficiency (3.2) is (with overbar indicating steady-state value)

$$c(t) = \overline{c} = \frac{N_{\text{bot}}}{N_s} - 1 .$$

This result shows that the retention efficiency can exceed one if the bottom-box nutrient concentration is more than twice that of the source water, $N_{\text{bot}} > 2N_s$. This “super-efficiency” is possible because nutrients that are retained in the nutrient-particle-oxygen cycle within the model boxes can effectively respire multiple times, rather than just once. An efficiency normalized instead by the upwelling nutrient flux into the top box, $\int \lambda N_{\text{bot}} dt'$, can be shown never to exceed one (except for a possible transient response to large initial particle loads), but is less useful because that flux is not known a priori from the imposed upwelling rate and source-water conditions. From (A.3), and (A.2), it is straightforward to see that the instantaneous depletion $\Delta O_{\text{bot}}$ is no greater than $\nu \int_0^t \lambda N_s dt'$ plus a constant, so the instantaneous depletion is limited by the total time-integrated onshore nutrient transport. In steady state, however, the latter becomes infinite while the depletion remains finite; thus, the associated alternative efficiency ratio of instantaneous depletion to total onshore transport will always approach zero, and so this alternative too is less useful than (3.2). On the other hand, (A.3) shows that oxygen depletion is proportional to enhancement of bottom nutrient concentration over its initial value,
which (neglecting the small initial nutrients above the bottom box) can only be achieved by onshore nutrient transport.

4. Rate controls on depletion amplitude and timescale

4.1 Asymptotic steady states

The case studies (Sec. 3) highlight two important characteristics of the temporal evolution of the steadily-forced model system: the timescale to reach a critical \( \Delta O_{\text{bot}} \) threshold, which is controlled by the initial growth phase in \( C_{\text{top}} \), and the longer term, asymptotically steady depletion that the system eventually approaches. The asymptotic values are steady states that can be determined analytically. These steady states are, of course, idealizations; ocean upwelling systems are always subject to some temporal variability. Further, as will be shown, the model steady-state depletion can be quite large, often exceeding the oxygen concentrations not only of typical upwelling source waters, but also exceeding that in the most oxygenated ocean waters. Nonetheless, the steady states can be used to characterize the maximum potential de-oxygenation over a wide range of parameter space. This allows analysis of the system’s response to key parameters, and of the factors that can drive the system to hypoxia and anoxia.

The steady states are approached when particle production eventually reaches nutrient limitation in the top box, slowing the growth of \( C_{\text{top}} \) and resulting, for a given value of \( s \), in a constant delivery of particles to the bottom box, in turn allowing a steady-state for \( \Delta O_{\text{bot}} \) that balances the flushing of the box by offshore water with consumption of oxygen by respiration of particles. Explicit expressions for these steady-state solutions can be obtained by setting the time derivatives in (2.4)–(2.8) to zero, and are given in the
Appendix [equations (A.4)–(A.9)]. The key result is for the steady-state oxygen depletion, $\Delta O_{bot}$:

$$\Delta O_{bot} = F(r, s, \lambda, \delta) \nu(N_s - N_{top}) \quad , \quad (4.1a)$$

where

$$F(r, s, \lambda, \delta) = \frac{\delta s^2}{rs + (r + s\delta)\lambda} \quad , \quad (4.1b)$$

and,

$$N_s - N_{top} = \left(1 - \frac{p_s K_s}{\alpha}\right)N_s = N_s - \left(\frac{p}{\alpha} - 1\right)K \quad . \quad (4.1c)$$

In (4.1c), $p_s = (r + s + \lambda)/N_s$, where $K_s = K/(K + N_s) \ll 1$ and $N_s = N_s/(K + N_s) = 1 - K_s \approx 1$ are dimensionless half-saturation constant and source nutrient levels, respectively. These steady states prove always to be stable: small perturbations from them decay, so that the perturbed solution evolves back toward the steady state at the exponential rate $\lambda r(\delta + \delta \Lambda)/[\delta s(r + \Lambda)]$.

The steady-state bottom oxygen depletion $\Delta O_{bot}$ is thus the product of two factors, with the first, $F$ defined in (4.1b), dependent only on respiration ($r$), sinking ($s, \delta$) and the physical forcing ($\lambda$). The nutrient source concentration $N_s$, half-saturation $K$, production rate $p$, and the stoichiometry $\nu$ appear only in the second factor, which additionally depends (through $\alpha$) on $r, s,$ and $\lambda$, but not on $\delta$. The last expression in (4.1c) shows directly that the dependence of $\Delta O_{bot}$ on $N_s$ is linear, so that an increase in source-water nutrient concentrations will cause a proportional increase in steady-state bottom oxygen depletion when all other parameters are held fixed.
If the production rate $p$ is sufficiently small that net particle growth is not
supported, then $\bar{N}_{top} = N_s$, and from (4.1a) there is no bottom oxygen depletion. When
the intrinsic surface particle production rate $p$ is large enough that production of $C_{top}$ is
supported (i.e., when $p$ is substantially greater than $p_*$), further increases in $p$ have almost
no effect on bottom oxygen depletion, provided that all other system parameters –
notably the sinking rate $s$ – are held fixed (Fig. 5a). Thus, in the productive regime, the
primary control on steady state bottom-oxygen depletion with respect to examined
variations in the model parameter space is exerted not by $p$ but by the first factor on the
right-hand side of (4.1a). This limited dependence on $p$ can be anticipated from the
second expression in (4.1c), as the first factor is nearly unity unless $\alpha \approx p, K_*$ [i.e., unless
$p \approx p_*$; see Appendix], and $\alpha = p - r - s - \lambda$ increases linearly with $p$. As $p$ decreases
(or as $r, s$, or $\lambda$ increases), the no-growth point is seen from (4.1c) to be first reached not
when $\alpha = 0$, but instead when $\alpha = p_*, K_*$ (or, equivalently, when $p = p_*$).

In contrast to variations in $p$, variations in $\lambda$ and $s$ have large effects on the
steady-state oxygen depletion (4.1) even far from the corresponding critical values
$\lambda_* = pN_* - r - s$ and $s_* = pN_* - r - \lambda$, where the no-growth point is again reached.

When $\lambda$ varies with all other parameters fixed at their values for the FP and SP particle-
specific parameters (Fig. 5b), the maximal oxygen depletion is found at minimal (zero)
upwelling forcing, and the minimum depletion (zero) at the corresponding critical
$\lambda$ values for these parameter sets ($\lambda_* = 0.37$ and $0.20$ d$^{-1}$, respectively). This behavior is
general: with $s$ and all other parameters fixed, both factors in (4.1a) decrease
monotonically with increasing $\lambda$ until $\Delta O_{bot}$ vanishes at the no-growth point, $\lambda = \lambda_*$. Thus, the maximum steady-state depletion is found as $\lambda$ approaches zero, and is equal to
achieves an hypoxic steady-state depletion over the allowable range of \( \lambda \), reaching a maximum of only 30 mmol m\(^{-3}\) even in the limit of zero upwelling forcing.

The approach to a maximum depletion for no upwelling \((\lambda = 0)\) is a curious result, suggesting that if upwelling never occurred, the system could reach maximum oxygen depletion. This is, of course, inaccurate: an unforced system has an oxygen depletion limited by the respiration potential of the initial \( C_{top} \) and \( N_{top} \) conditions, as can be verified directly. For the steady-state solutions, however, oxygen depletion increases with decreasing upwelling all the way to the limit of zero upwelling. The resolution of this apparent inconsistency comes in the very long times necessary to achieve the steady state under infinitesimal upwelling forcing: infinitesimal upwelling produces maximal oxygen depletion, but only after infinitely long timescales.

The steady-state solutions also show a strong dependence on the sinking rate \( s \) (Fig. 5c). Depletion increases with \( s \) over most of the allowable range, in accordance with the \( s \)-dependence of the first factor in (4.1a), as increasing the sinking rate enhances the export of \( C_{top} \) and reduces attenuation by respiration in the middle box. A sharp cut-off occurs for the largest allowable \( s \), as fast sinking rates remove surface particles so rapidly that production declines and less nutrients are consumed. The minimum sinking rate is trivially 0; the maximum sinking rate, analogous to the maximum upwelling rate, comes when \( s = s_* \), making the net \( C_{top} \) accumulation rate zero and extinguishing export. In between these two limits, and close to the upper limit \( s = s_* \), there is an optimum value \( s_{opt} \) that maximizes oxygen depletion potential. Steady-state de-oxygenation is thus bounded by \( 0 < \Delta O_{bot} < F(r, s_{opt}, \lambda, \delta) \nu (1 - p_r K_r / \alpha_{opt}) N_s \), where the lower limit
corresponds to \( s = 0 \), the upper limit to \( s = \alpha_{opt} \), and \( \alpha_{opt} = p - r - s_{opt} - \lambda \); an analytical expression for \( s_{opt} \) is derived below (Sec. 5.2).

With other parameters fixed as in the FP and SP cases (Fig. 5c), the optimal sinking rates \( s_{opt} \) are found near \( s = 0.65 \) and \( 0.13 \, \text{d}^{-1} \), respectively, close to the corresponding cut-off values, \( s_* = 0.77 \) and \( 0.15 \, \text{d}^{-1} \). The maxima around these optimal sinking rates are broad: oxygen depletions of up to half the maximum are found over roughly half the allowable ranges of \( s \). The depletions for the FP and SP optimal sinking rates are very different: with a modified, near-optimal sinking rate, FP can attain extreme hypoxia (\( \Delta O_{bot} > 100 \, \text{mmol m}^{-3} \)), while even the optimal sinking rate for SP results in only a modest depletion (17 mmol m\(^{-3} \)). For small \( s \), however — that is, below the cut-off value \( s_* \) for SP — the depletions are essentially identical, indicating that the different values of \( p \) and \( K \) for FP and SP have almost no effect on the relative oxygen depletion levels. Instead, it is the different values of \( s \) that control the behavior; the larger \( p \) value for FP is important as a driver of potential hypoxia only insofar as it allows larger values of \( s \) without extinguishing surface productivity.

4.2 Timescales to hypoxia

As noted above, the initial surface particle response is largely not nutrient-limited, and approximately follows an exponential growth trajectory until \( N_{top} \) is sufficiently depleted to limit \( C_{top} \) growth. Since oxygen depletion is driven by respiration of particles in the bottom box, which are ultimately supplied by the export of particles from the top box, the exponential rise in \( C_{top} \) has direct bearing on the initial rate of depletion. For this exponential growth phase, the model surface-box \( C_{top} \) has an approximate analytical solution given by:
This shows the leading dependence of the initial response on the model parameters in the form of the previously defined maximal accumulation rate $\alpha = p \cdot r \cdot s \cdot \lambda$.

The exponential approximation yields reasonable agreement with the full solution for $C_{\text{top}}$, within a factor of two for the FP case for $\sim 10$ days, and within a factor of 1.5 for the SP case all the way to the $C_{\text{top}}$ maximum at 56 days (Figures 2b, 3b), but the greater relevance is to the initial rise in $\Delta O_{\text{bot}}$. The existence of the approximate analytical solution (4.2) for $C_{\text{top}}$ while $N_{\text{top}} \gg K$ leads to a similar approximation for $\Delta O_{\text{bot}}$:

$$\Delta O_{\text{bot}}(t) \approx \frac{\nu C_{\text{top},0}}{r G(r, s, \lambda, \delta)} e^{\alpha t}$$

In (4.3), the function $G$ is

$$G(r, s, \lambda, \delta) = \frac{(\alpha + \lambda)(\alpha + r)(\alpha + r + \delta s)}{r \delta s^2}$$

and captures the factors that ultimately link export of $C_{\text{top}}$ with input to, and respiration in, the bottom box. Comparison of the full solution for $\Delta O_{\text{bot}}$ with the exponential approximations shows remarkable adherence for a surprisingly long time: For the FP case, the exponential approximation is within a factor of two of the full solution for 15 days (Fig. 2a), while for the SP case the agreement is within 20% of the smaller $\Delta O_{\text{bot}}$ for over 50 days (Fig. 3a). In the FP case, where hypoxic thresholds are achieved, the exponential approximation finds moderate hypoxia ($\Delta O_{\text{bot}} = 40 \text{ mmol m}^{-3}$) at 18 days, just 4 days before that condition is attained by the full model, suggesting a strong predictive capability for the onset of hypoxic conditions using this simplified approximation and knowledge of a handful of inherent system timescales. Predictions beyond that point are worse but still meaningful, with the exponential approximation predicting extreme hypoxia in 21 days, while the full solution requires 31 days to reach
Equation (4.3) can be rearranged to solve for the time \( t_{hyp} \) at which a specific value of \( \Delta O_{bot} \), \( \Delta O_{hyp} \), is reached:

\[
t_{hyp} \approx \frac{1}{\alpha} \ln \left( \frac{y\Delta O_{hyp}}{\nu C_{top0} G} \right).
\]  

As the SP case never results in appreciable oxygen depletion, the timescales to hypoxia for the exponential approximations and the full numerical solutions are compared for varying \( \lambda \) (Figure 6a) and \( s \) (Figure 6b) only for the nominal FP case with the fast growth rate (large \( p \)). The full solutions predict broad minima of \(< 30\) days for \( \Delta O_{bot} = 40\) mmol m\(^{-3}\), for \( 0.02 < \lambda < 0.2 \) (flushing timescale of 50 - 5 days) with such conditions reached in as little as 20 days for \( \lambda \approx 0.04 \), or a flushing timescale of 20 days. The times to reach extreme oxygen depletion (\( \Delta O_{bot} = 100\) mmol m\(^{-3}\)) are correspondingly greater, with this condition being reached within 40 days for \( 0.04\) d\(^{-1} < \lambda < 0.15\) d\(^{-1}\) (25 - 7 d flushing timescale), with a fastest attainment of 30 days at \( \lambda \approx 0.08 \) (12 day upwelling timescale).

Sensitivity to \( s \) shows qualitatively similar patterns: There are broad minima in the attainment of \( \Delta O_{bot} = 40\) mmol m\(^{-3}\) (<30 days for \( 0.2\) d\(^{-1} < s < 0.6\) d\(^{-1}\), with a minimum of 20 d for \( s \approx 0.4\) d\(^{-1}\)) and \( \Delta O_{bot} = 100\) mmol m\(^{-3}\) (<40 days for \( 0.35\) d\(^{-1} < s < 0.6\) d\(^{-1}\), with a minimum of 30 days for \( s \approx 0.35\) d\(^{-1}\)). Outside of these respective minima, the timescales for achieving significant oxygen depletion increase rapidly, suggesting that outside of these parameter bounds there is limited capacity for achievement of hypoxia within an upwelling season, regardless of the asymptotic potential oxygen depletion.

The exponential approximations provide reasonable estimates of the full-solution timescales within these broad minima, although they are consistently lower. For the
ΔO_{bot} = 40 \text{ mmol m}^{-3} \text{ case, the exponential approximation (4.5) yields timescales that are}
within 75\% of those predicted from the full model, and within 60\% for that minimum for
the ΔO_{bot} = 100 \text{ mmol m}^{-3} \text{ case. For varying } s, \text{ the comparison is slightly worse, with}
exponential approximations as low as 50\% of the full model solution for the 40 \text{ mmol m}^{-3}
case and 40\% for the 100 \text{ mmol m}^{-3} \text{ case. Nonetheless, the exponential approximations}
give predictions that are mostly within a factor of two, and often much better, of the
timescales of bottom oxygen depletion for wide ranges of parameter space.

4.3. Control by production and sinking rates: a combined perspective.

For a given set of model parameters, the timescale (4.5) for approach to hypoxia
is only relevant if the steady-state depletion (4.1) is large enough to make the system
potentially susceptible to hypoxia. If the steady state depletion (4.1) is sufficient to cause
hypoxia, then the exponential approximation (4.5) will provide an estimate of the
timescale to reach hypoxia that is accurate enough to evaluate, for example, whether
hypoxia might be reached within an upwelling season. If, on the other hand, the steady
state depletion is not sufficient to cause hypoxia, then the timescale estimate (4.5) does
not apply, because the system will never reach hypoxia. The steady-state depletion (4.1)
depends strongly on sinking rate \( s \), but is nearly independent of production rate \( p \) as long
as a threshold rate for growth is exceeded (Fig. 5). In contrast, the estimated timescale to
hypoxia (4.5) depends strongly on \( p \) and only weakly on \( s \) (Fig. 6). A combined
perspective on the susceptibility of the model system to hypoxia can be developed by
considering the dependence of these two metrics jointly on \( p \) and \( s \), with other parameters
held fixed.
The combined perspective can be efficiently described, or assisted, by a single contour plot of the depletion (4.1) and timescale (4.5) vs. $p$ and $s$ (Fig. 7), which shows only the relevant one of these two quantities in each sector of the plot: only the timescale, if the steady-state depletion is hypoxic, and only the steady-state depletion, if this depletion is not hypoxic. Thus, for a given hypoxic threshold $\Delta O_{hyp}$ (chosen for Fig. 7 as the extreme value $100$ mmol m$^{-3}$, corresponding to anoxia when $O_s = 100$ mmol m$^{-3}$), contours of the exponential timescale (4.5) are drawn where the steady state depletion (4.1) exceeds $\Delta O_{hyp}$, while contours of the depletion (4.1) itself are drawn where its value does not exceed $\Delta O_{hyp}$. The resulting combined contour plots show more generally, for a broad range of $p$ and $s$ values, that the ultimate capacity for oxygen depletion is mostly dependent on $s$, and is essentially independent of $p$ values, except near the critical line $s = s_c (\alpha = pK_s)$. In contrast, the timescales for attaining hypoxic states are mostly sensitive to $p$, and are nearly independent of $s$.

The similarity of these representations for the FP (Fig. 7a) and SP (Fig. 7b) values of $K$ shows additionally that the difference in $K$ has negligible effect on the potential for oxygen depletion. The ultimate development of hypoxia is thus entirely driven by coincidence of two fast rates: high sinking rates ($s > 0.3$ d$^{-1}$) are required for development of consequential hypoxia, while high surface production rates ($p$ greater than the critical value $\alpha/K_s$) are required for that development to take place on reasonable timescales relative to an upwelling season.
5. Generalized analysis of susceptibility to hypoxia

5.1 Relative-rate scaling

Even in this highly simplified model, the number of free parameters is large enough to confound a complete characterization of the oxygen depletion response as a function of all model parameters. However, substantial progress toward this goal is possible. For a given system, the combined analysis of steady-state depletion and exponential timescale developed above (Sec. 4.3, Fig. 7) provides a relatively complete description of the susceptibility to hypoxia for all plausible values of \( p \) and \( s \). Further, the steady-state depletion (4.1) was seen to be linearly dependent on \( N_s \), while the timescale to hypoxia, approximated following (4.5) under the assumption \( N_s \gg K \), is independent of \( N_s \).

A first step toward a more general characterization follows from the recognition that the steady-state depletion (4.1) depends only on the dimensionless ratios of the fundamental rates, not their absolute, dimensional values. Consequently, choosing one rate to define a master timescale, and normalizing the others by that master rate can remove one free parameter. The sum of the respiration and upwelling rates, \( r + \lambda \), describes the non-sinking fraction of surface particle loss, and proves to be a convenient choice for the master rate. With this choice, the normalized particle production, sinking, and maximal accumulation rates are

\[
(\tilde{p}, \tilde{s}, \tilde{\alpha}) = (p, s, \alpha)/(r + \lambda) \quad , \quad (5.1)
\]

where \( \tilde{\alpha} = \tilde{p} - 1 - \tilde{s} \), and the normalized sinking rate \( \tilde{s} \) represents the ratio of sinking to non-sinking fractions of surface particle loss. The steady-state depletion (4.1) can then be written as,
\[ \Delta O_{bot} = \bar{F}(\bar{s}, \delta, \Lambda) \nu \left[ N_s + \left( 1 - \frac{\bar{p}}{\bar{a}} \right) K \right] \quad , \quad (5.2a) \]

where

\[ \bar{F}(\bar{s}, \delta, \Lambda) = \frac{\delta (1 + \Lambda)^2 \bar{s}^2}{\Lambda + (1 + \Lambda)(1 + \delta \Lambda) \bar{s}} \quad , \quad (5.2b) \]

and

\[ \Lambda = \lambda/r \quad \quad (5.2c) \]

is the ratio of upwelling rate to respiration rate. Thus, \( \Delta O_{bot} \) is seen from (5.2) to depend on only three dimensionless, relative rate parameters – \( \bar{s}, \Lambda, \) and \( \bar{p} \) – rather than the four dimensional rate parameters \( p, r, s, \) and \( \lambda \) in (4.1). Similarly, the scaled exponential depletion timescale \( \tilde{t}_{hyp} = (r + \lambda) t_{hyp} \) can be seen from (4.4)-(4.5) also to be a function only of the relative rates.

This reduction to dependence on relative rates means that, when other parameters are fixed, the system response (including the dimensionless timescale \( \tilde{t}_{hyp} \)) is unchanged if the dimensional production, sinking, respiration, and upwelling rates are all increased or decreased by the same factor. Note that this implies that the dimensional timescale

\[ t_{hyp} = \tilde{t}_{hyp}/(r + \lambda) \]

is changed by the inverse of that same factor: if all rates were doubled, for example, the steady-state depletion would be unchanged, but the dimensional timescale \( t_{hyp} \) would decrease by half.

When the combined depletion and timescale analysis (Sec. 4.3, Fig. 7) is recast and expressed in terms of the relative production and sinking rates, \( \bar{p} \) and \( \bar{s} \), it is seen to apply much more generally to all systems with the same values of these relative rates, rather than only for a single value of \( r \) and a single value of \( \lambda \) (such as the FP and SP values \( r = 0.2 \text{ d}^{-1}, \lambda = 0.1 \text{ d}^{-1} \), for Fig. 7). The system dependence on \( K \) is more evident if
the relative maximal accumulation rate $\bar{\alpha} = \bar{\alpha} - 1 - \bar{s}$ is used in place of $\bar{\alpha}$ in this context (Fig. 8). This description in terms of the relative rate parameters $\bar{\alpha}$ and $\bar{s}$ is valid for all systems that have the same ratio of upwelling to respiration rates (fixed $\Lambda = \lambda/r$) and, of course, the same values of the other relevant fixed parameters ($\delta, K, N_s$, and, for $t_{hyp}$, the initial surface particle concentration $C_{top0}$ and depletion threshold $\Delta O_{hyp}$).

It was seen above (Fig. 7) that, when other parameters are fixed as in the FP or SP examples, the system was susceptible to hypoxia (or even anoxia) when $s > 0.3 \text{ d}^{-1}$. The relative-rate analysis shows that this criterion corresponds more broadly to a dimensionless sinking rate $\bar{s} > 1$ (Fig. 8): setting $\bar{s} = 1$ in (5.2), with $\delta = 1/3$ and $\Lambda = 1/2$ as in FP or SP, gives (above the productivity threshold, i.e., for $p > p_*$) $\Delta O_{bot}(\bar{s} = 1) \approx \delta v N_s \approx 117 \text{ mmol m}^{-3}$. (5.2)

Similar steady-state depletions will be obtained for similar values of $\delta$ and $\Lambda$, with the analytical approximation (5.2) holding whenever $\delta \approx \Lambda/(1 + \Lambda)$ and $\Lambda$ is not too large.

This result, like that for $\Delta O_{bot}(\lambda \to 0)$ in Sec. 4.1, also shows the potentially strong effect of variations in the parameter $\delta$.

An important general conclusion thus follows from this relative-rate analysis: with $\Lambda$ and other parameters fixed as in the FP or SP examples, the system will be susceptible to hypoxia whenever the sinking rate is comparable to or greater than the sum of the respiration and upwelling rates ($\bar{s} \geq 1$, so that at least half of the surface loss is to sinking), provided also that the production rate is sufficiently large to support growth at the given sinking rate. If, on the other hand, the sinking rate is much smaller than the sum of the respiration and upwelling rates (as in the SP example with the SP value of
sinking rate \( s = 0.05 \, \text{d}^{-1} \ll r + \lambda = 0.3 \, \text{d}^{-1} \), then the system can never reach anoxia, even for arbitrarily long upwelling seasons.

Typically, the dimensionless exponential timescale \( \tilde{t}_{hyp} < 10 \) for \( \tilde{\alpha} < 0.25 \) (Fig. 6), implying that if the dimensional maximal accumulation rate \( \alpha = p - r - s - \lambda \) is at least one-fourth the sum \( r + \lambda \) of the respiration and upwelling rates, the dimensional time to hypoxia will be of order \( 10/(r + \lambda) \, \text{d} \) or less. The latter will in turn be less than 100 d if the sum of the respiration and upwelling rates is greater than 0.1 d\(^{-1}\). This implies that hypoxia can be reached within the upwelling season even with relatively small maximal accumulation rates that lead to little \( C_{top} \) accumulation (e.g., for \( \alpha \approx 0.03 \, \text{d}^{-1} \)), if sinking rates are large enough.

5.2 Hypoxic-optimal sinking

Even after the relative-rate scaling (Sec. 5.1), the summary results (e.g., Fig. 8) still depend on several potentially important parameters, especially \( \delta, \Lambda \) and \( K \) or \( K_* \), all of which must be held fixed for that analysis. It was seen in the FP and SP examples that the steady-state oxygen depletion, over much of the range of allowable values of sinking rate \( s \), could be roughly characterized by the depletion at the optimal sinking rate \( s_{opt} \), for which depletion is maximized (Sec. 4.1, Fig. 5b): the depletion is generally half or more of the maximal value over roughly half of the full range \( 0 < s < s_* \) of sinking rates that support particle concentration growth. For example, when \( s \) is varied while other parameters are held fixed as in the FP and SP cases, depletions of over 100 mmol m\(^{-3}\) and over 15 mmol m\(^{-3}\), respectively, roughly half the corresponding maximal values, are obtained throughout the respective ranges \( 0.25 < s < 0.75 \) and \( 0.06 < s < 0.13 \) (Fig. 5b).

Restricting attention to this hypoxic-optimal sinking rate, and to the depletion at this rate,
removes the dependence on sinking rate and thereby again reduces the number of free
model parameters by one.

The hypoxic-optimal sinking-rate definition may be further generalized by using
the approach in Section 5.1 to obtain its dimensionless relative-rate equivalent, \( \tilde{s}_{opt} = \frac{s_{opt}}{r + \lambda} \). Both this scaled sinking rate \( \tilde{s}_{opt} \) (Fig. 9a) and the corresponding maximal
accumulation rate \( \tilde{a}_{opt} = p - 1 - \tilde{s}_{opt} \) (Fig. 9b) increase monotonically with \( p \) for any
value of \( K_* \). The dependencies of \( \tilde{s}_{opt} \) and \( \tilde{a}_{opt} \) on \( K_* \) are opposite, with the former (Fig.
9a) decreasing and the latter (Fig. 9b) increasing with \( K_* \). Thus, a system with smaller \( K_* \)
will support a larger optimal sinking rate, at a smaller corresponding maximal
accumulation rate, than an otherwise identical system with larger \( K_* \). The primary
dependence revealed by this analysis, however, is the indirect control of oxygen depletion
potential by the production rate \( p \) (or \( \tilde{p} \)), through its effect on allowable sinking rates. The
near linear increase of \( \tilde{s}_{opt} \) with \( \tilde{p} \) (Fig. 9a) illustrates again the primary role that intrinsic
productivity plays in controlling the system’s susceptibility to hypoxia: it sets the
maximum sinking velocity that can be supported without shutting down production.

An accurate analytical approximation is available for \( s_{opt} \) as a function of
production rate \( p \) (see Appendix). In relative-rate form, this analytical approximation is
\[
\tilde{s}_{opt} \approx \tilde{s}_{max} \left[ 1 - \left( \tilde{p}K_* / \tilde{s}_{max} \right)^{1/2} \right] = \tilde{p} - 1 - \left[ (\tilde{p} - 1)\tilde{p}K_* \right]^{1/2}, \quad (5.4)
\]
where \( \tilde{s}_{max} = \tilde{p} - 1 \) is the sinking rate for which the maximal accumulation rate
vanishes (i.e., for which \( \tilde{a} = 0 \)). Remarkably, the approximate optimal sinking rate (5.4)
depends only on \( \tilde{p} \) and \( K_* \), and is independent of all other parameters, including \( \delta \) and \( \Lambda \).
The corresponding relative maximal accumulation rate, which the system would exhibit
for sinking at the optimal rate, is
\[ \tilde{\alpha}_{opt} = \bar{p} - 1 - \tilde{s}_{opt} = [(\bar{p} - 1)\bar{p}K_*]^{1/2}. \quad (5.5) \]

In general, \( K_* \ll 1 \), so the optimal sinking rate (5.4) is always close to the \( \tilde{s}_{max} \), the upper bound for net surface accumulation.

### 5.3 Dependence on \( \delta \) and \( \Lambda \)

As anticipated (Sec. 4.1), the optimal sinking rate \( \tilde{s}_{opt} \) from (5.4) can be substituted into (4.1) to obtain the corresponding maximal steady-state oxygen depletion \( \Delta O_{bot}(\tilde{s}_{opt}) \) that is possible for given values of \( \bar{p}, K_* \), and the other fixed parameters.

With the dependence on \( s \) removed in this way, attention can be focused on the dependence of the steady-state response on the previously fixed sinking-rate and upwelling-to-respiration-rate ratios, \( \delta \) and \( \Lambda \).

For the FP values of \( \bar{p} \) and \( K_* \), specific-sinking-rate ratios \( \delta > 0.5 \) give uniformly large steady-state oxygen depletions \( \Delta O_{bot}(\tilde{s}_{opt}) \), for a wide range of upwelling to respiration rate ratios \( \Lambda \), while small optimal-sinking oxygen depletions are obtained only for small \( \delta \) (Fig. 10a). The corresponding exponential timescales are all short, and shorter for small \( \Lambda \) than for large \( \Lambda \) (Fig. 11a). For the SP values of \( \bar{p} \) and \( K_* \), ratios \( \delta > 1.0 \) give potentially anoxic steady-state depletions, especially for small \( \Lambda \) (Fig. 10b). Thus, in principle, the SP values could support hypoxia for a thin middle box \( (H_{mid} \leq H) \) or rapid deep sinking \( (S' >> S) \). However, the corresponding exponential timescales (Fig. 11b) are all relatively long \( (t_{hyp} > 45) \); for \( r + \lambda = 0.3 \text{ d}^{-1} \), for example, the dimensional timescales are longer than the nominal 150-day upwelling season length. The overall differences in these maximal FP and SP depletions are primarily the result of the different values of \( \bar{p} \), not \( K_* \), for the two cases.
5.4 General criteria for hypoxic susceptibility

For the steady states, which provide a basic characterization of the system’s potential susceptibility to hypoxia, the relative-rate and optimal-sinking analyses effectively remove two of the six independent parameters that remain after the proportionality of the steady-state depletion $\Delta O_{bot}$ to $N_s$ is recognized, leaving the four parameters $\bar{p}, \delta, \Lambda$ and $K_r$. If the dependence of the maximal oxygen depletion $\Delta O_{bot}(\bar{s}_{opt})$ on $\Lambda$ is then examined, two simple criteria on $\bar{p}$ emerge that broadly express the susceptibility to hypoxia as functions of the two remaining parameters, $\delta$ and $K_r$. Because each of these criteria depend on only two parameters, the dependence of each can be completely described by a single contour plot (Fig. 12). The result is a compact and comprehensive description of the intrinsic susceptibility to hypoxia of the full model system, which assumes only that the steady-state bottom-oxygen depletion may be characterized by that obtained at the optimal sinking rate, or, equivalently, that the model sinking rate is of the same order as the optimal rate. The question of timescale to attain hypoxia is not addressed by this analysis and must be considered separately.

The criteria on $\bar{p}$ show that the susceptibility to hypoxia depends (see Appendix) on the values of two constants, $P(\delta, K_r)$ and $P'(\delta, K_r)$, relative to the dimensionless production rate $\bar{p}$, which as seen before is just the ratio of the dimensional production rate to the sum of the dimensional respiration and upwelling rates. For a given value $\Delta O_{hyp}$ that is specified as the hypoxic level, these constants divide the steady-state response into three regimes (Fig. 9): if $\bar{p} < P$, then the system will never be hypoxic; if $P < \bar{p} < P'$,
then hypoxic steady states can occur for some sinking rates and some values of the ratio \( \Lambda = \lambda / r \) of upwelling and respiration timescales; and if \( \bar{P} > P' \), then all steady states with normalized sinking rate equal to the optimal rate \( \bar{s}_{opt} \) are hypoxic, regardless of the value of \( \Lambda \). These criteria may be expressed also in terms of the corresponding critical values \( P_{\bar{a}}(\delta, K_\ast) \) and \( P'_{\bar{a}}(\delta, K_\ast) \) of \( \bar{a} \), where the latter are obtained by substituting \( P \) and \( P' \), respectively, for \( \bar{P} \) in (5.5).

The form of these criteria indicate that the potential for large steady-state oxygen depletion is greater for systems with relatively rapid production (large \( p \)) and relatively slow respiration and upwelling (small \( r \) and \( \lambda \)), which will in turn yield relatively large normalized production rates \( \bar{p} \) that will be more likely to satisfy the inequalities \( \bar{P} > P \) and \( \bar{P} > P' \). This may appear to contradict the previous results (e.g., Figs. 5a, 7, 8) that showed the relative unimportance of the production rate \( p \) for the steady-state depletion, which was seen instead to depend strongly on the sinking rate \( s \). The apparent paradox arises from the assumption of optimal sinking (5.4) that is used to derive the criteria. This assumption effectively couples the production and sinking rates, so that a larger sinking rate is implicitly associated with a larger production rate. Under this assumption, the previous indirect role of the production rate, which was found to be important only insofar as larger values allowed larger sinking rates without shutting down production, becomes a direct, controlling role.

For \( \Delta O_{hyp} = 100 \text{ mmol m}^{-3} \), the values of \( (P, P') \) are (1.54, 2.39) for FP and (1.40, 2.12) for SP, while the values of \( \bar{P} \) are 3.67 for FP and 1.50 for SP (Table 3). Thus, the stricter inequality \( \bar{P} > P' \) is easily satisfied for the FP values, while the weaker inequality \( \bar{P} > P \) is only narrowly satisfied for the SP values (Fig. 9). Although
neither the SP nor the FP value of the sinking rate $s$ is equal to the corresponding optimal rate $\delta_{opt}$, it can nonetheless be anticipated from these inequalities that the steady-state depletion for FP is likely to be sufficient to produce an anoxic steady state (for $O_9 = 100$ mmol m$^{-3}$), while that for SP is much less likely to be sufficient to produce anoxia.

Consideration of the dependence of $P$ and $P'$ on $\delta$ and $K_*$ yields, finally, the desired comprehensive description of the hypoxic susceptibility of the full model system, based on the steady states and under the relatively weak condition that the source-water nutrient concentration $N_x$ is fixed. In general, the potential for hypoxia increases as the constants $P$ and $P'$ decrease toward unity, reducing the value of $\bar{p}$ that would be needed to satisfy the corresponding inequalities. The minimal value of $\bar{p}$ for steady-state growth is $\bar{p} = \bar{p}_* = 1/N_x \approx 1$, so when $P = 1$ or $P' = 1$, all values of $\bar{p}$ that support growth will satisfy the corresponding criteria. Both constants increase everywhere as $\delta$ and as $K_*$ increase (Fig. 12). Thus, larger specific-sinking-rate ratios $\delta$ and smaller half-saturation constants $K_*$ favor the development of hypoxia.

The dependence of two constants on $\delta$ and $K_*$, while monotonic, is not linear (Fig. 12). For both constants, the dependence on $K_*$ is generally stronger for larger $\delta$ than for smaller $\delta$ (Fig. 12). The smaller constant, $P$, which determines the production rate below which no hypoxic steady states occur, has a distinct change in dependence at $\delta = 1$: for $\delta < 1$, $P$ is independent of $\delta$, while for $\delta > 1$, $P$ decreases rapidly with $\delta$. The larger constant, $P'$, has a contrasting behavior, with a stronger dependence on $\delta$ for $\delta < 1$, and a smooth transition between the two regimes.

6. Discussion
The preceding analysis, while based on a highly idealized model, can be used to make rough estimates of hypoxic susceptibility for any upwelling system for which approximate values for the basic model parameters are available, as follows:

- From values, or ranges of values, of the relevant half-saturation constant $K$, the source-water nutrient concentration $N_s$, and the specific-sinking-rate ratio $\delta = s'/s = (S'/H_{mid})/(S/H)$, the values of the constants $P$ and $P'$ can be computed for a specific hypoxic threshold $\Delta O_{hyp}$ (Fig. 12 or equations in Appendix). Hypoxia will generally be favored if these constants are close to one, and less favored if they are larger than one.

- From values, or ranges of values, of the production, respiration, and upwelling rates, the dimensionless production rate $\bar{p} = p/(r + \lambda)$ can be computed, and compared to the constants $P$ and $P'$ to provide a rough classification of the system as (1) likely, (2) able, or (3) unable to support hypoxia, where these cases correspond respectively to (1) $\bar{p} > P'$, (2) $P < \bar{p} < P'$, and (3) $\bar{p} < P$. The sinking rate can be compared to the optimal rate (5.4); if it is much less than the optimal rate, then the classification will be conservative, and the susceptibility to hypoxia will be less than indicated.

- From values, or ranges of values, of the production, respiration, sinking, and upwelling rates, as well as the sinking-rate ratio and the initial surface particle concentration, the maximal accumulation rate $\alpha = p - r - s - \lambda$ and the associated timescale $t_{hyp}$ (4.4) for the onset of hypoxia can be computed. The timescale $t_{hyp}$ can then be compared to the length of the relevant upwelling season, to estimate the likelihood that systems are potentially –
based on comparisons of dimensionless production rate $\bar{p}$ with the constants $P$ and $P'$ – able to support steady-state hypoxia will indeed be able to reach hypoxia seasonally.

The model, as formulated here, predicts rapid and extensive oxygen depletion across vast swaths of parameter space, encompassing many combinations typical of upwelling systems. When it does not predict hypoxia, it often exhibits unrealistic particle distributions, with far more accumulation in the surface than is ever observed and little accumulation at depth. These results therefore strongly suggest that most observed real-world systems have strong tendencies toward hypoxia, and the question then evolves from what controls the timing and extent of hypoxia to why hypoxia is not an even more prevalent feature of these systems. For the above analysis and considerations, the upwelling rate has been fixed in time. In many upwelling systems, however, the rate and intensity of upwelling varies strongly in time (Carr 1998; Perlin et al., 2005; Bane et al. 2007). Hales et al. (2006) suggested that this modulation of the upwelling forcing presented an export pathway for the missing particle production over the Oregon shelf that should or would have led to far greater oxygen depletion than observed.

Extensions of the present analysis, in which an oscillatory component is added to the upwelling rate, so that the system fluctuates between strong and weak or vanishing upwelling, indicate that time-dependent forcing can significantly reduce the potential for oxygen depletion, relative to constant forcing cases with equivalent cumulative or time-mean forcing. For example, if the forcing in the FP case is modified to

$$\lambda'(t) = \lambda(1 - \cos \omega t), \quad (6.1)$$
so that the upwelling rate oscillates between $2\lambda$ and zero, but the mean forcing over an upwelling period is still $\lambda$, then the mean oxygen depletion is reduced by over 50 mmol m$^{-3}$ relative to the steady-state value with constant forcing at rate $\lambda$ (Fig. 13). This reduction is accomplished via lowered retention efficiency (Eq. 3.2; Fig. 14a), resulting from increased offshore transport of surface nutrients and/or particles, which reduces sinking export over the shelf and so also bottom oxygen depletion. This offshore transport occurs when there is a correlation of strong forcing with either high surface nutrient or particle levels. Figure 14b clearly shows, however, that the greatest loss of potential respiration is in the offshore transport of $N_{top}$ during the maximal upwelling, which is much greater than for the constant-forcing case while offshore export of $C_{top}$ is much less. The mechanism therefore is that the productivity cannot keep up with the fastest upwelling rates, and $N_{top}$ is lost from the system before $C_{top}$ is produced. There have been direct observations of this scenario. Evans et al. (2015) reported time-series data of surface signatures of productivity in response to a particularly intense upwelling event, and showed that the productivity response could not keep pace with the most intense upwelling. A more detailed study of this effect and the associated processes in this simple model is in progress and will be reported on separately.

Some conclusions about what these results may mean for the future as well as for present-day spatial variability in hypoxia can be drawn from the analysis presented here. The typical period of oscillation of winds in the northern California Current System (CCS) is 8 days (Austin and Barth, 2002), and has been suggested both to be getting longer and more intense (Iles et al., 2012) or, alternatively, to have no trend (Bylhouver et al., 2013). The productivity response to increased upwelling seen here is consistent
with the suggestions of Bakun et al., (2015) and the observations of Evans et al. (2015),
in that there is an ideal window or an optimal environmental range for productivity that
results from a balance between the timescales of upwelled and offshore transport of
nutrient and those of the autotrophic response. Our results build on that and suggest a
negative feedback between increased upwelling and hypoxia, for other parameters
constant. Faster upwelling leads to less retention of respiration potential, and greater
export of this potential to the open ocean where it can be more broadly laterally and
vertically dispersed rather than being focused on shelf bottom waters.

For representative parameters and steady forcing, the predicted timescale for
reaching mild hypoxia ($\Delta O_{bot} = 40 \text{ mmol m}^{-3}$) is about 20 days (Section 4.2; Fig. 6),
based on current oxygen levels in the source waters for the CCS. Those waters have been
declining in oxygen at a rate of approximately 1 mmol m$^{-3}$ yr$^{-1}$ (Bograd et al., 2015;
Crawford and Pena, 2013), and if that trend continues, the mild hypoxia threshold defined
here will instead approach the severe threshold in about 40 years, and specific $O_{bot}$
criteria will be reached with even more rapidity. Along with this drop in $O$, is a likely
stoichiometric increase in $N_s$, which further amplifies this trend. Even if neither the mean
wind forcing nor its modulation change significantly, more frequent severe hypoxic
events can be anticipated in the future.

In addition, insights into mechanisms responsible for generating the spatial
variability along the coast in the CCS can be explained, in part, by the analysis presented
here. The wider shelves and weaker upwelling winds present in the northern part of the
California Current System, for example, would reduce the upwelling forcing, allowing
greater productivity response and retention of respiration in near-bottom waters over the
shelf. The northern CCS experiences an increased occurrence of hypoxia relative to the south (Peterson et al., 2013), which is consistent with the shorter oxygen drawdown times found there (Figure 6). In contrast, the winds are stronger and the shelves narrower farther south in the CCS, which is potentially what makes that region less susceptible to the respiratory potential and influenced more by the flushing of low-oxygen water from offshore.

7. Summary

A maximally simple, coupled physical-biogeochemical box model has been used to examine oxygen depletion near the shelf bottom in ocean-margin upwelling systems. It was found possible to achieve a relatively complete characterization of the bottom oxygen depletion response in this model. The major biogeochemical control on the overall potential susceptibility of the system was found to be the sinking rate $s$: for sufficiently small sinking rates, the system can never reach hypoxia even for arbitrarily long upwelling seasons, while the critical sinking rate allowing the system to approach hypoxia is nearly independent of the production rate $p$ and maximal accumulation rate $\alpha = p - r - s - \lambda$. For sinking rates large enough that hypoxia could in principle be reached, the maximal accumulation rate was found to be the major control on the timescale of approach to hypoxia: for sufficiently large maximal accumulation rates, hypoxia can be reached in typical upwelling seasons, while for very small maximal accumulation rates, the time to hypoxia can exceed the length of typical upwelling seasons, and hypoxia – although possible in principle – cannot be reached.

The primary influence of the particle production rate on hypoxia in this model is indirect: larger production rates allow growth at larger sinking rates, and larger sinking
rates increase oxygen depletion. Sinking velocities are poorly constrained by observations; however, if the sinking rate is presumed to be near the optimal rate for producing hypoxia, then the constraints on sinking rate translate into constraints on the production rate. Observations in upwelling systems of primary production are more prevalent than those of sinking rates and can even be garnered from satellite products. If rough estimates of the relevant rates and parameters are available for a given upwelling system, these can be used with the model results to obtain a basic characterization of the system’s potential susceptibility to hypoxia that requires minimal computation (Sec. 4).

In general, oxygen depletion in this model decreases with stronger upwelling forcing (Figs. 5, 7). To support hypoxia, the sinking rate must generally be comparable to the sum of the respiration and upwelling rates (Fig. 8), with the precise relation depending on the sinking-rate ratio \( \delta \) and other factors. Thus, for large upwelling rates, which can arise either from strong wind forcing or narrow shelf width, sinking must be rapid to support hypoxia, even for arbitrarily long upwelling seasons. Additionally, if hypoxia can occur and the system is forced sufficiently to overcome nutrient limitation, then the timescale for approach to hypoxia lengthens with stronger forcing, since particle production is shunted off-shelf and surface particle accumulation decreases, in turn reducing subsurface respiration and oxygen depletion. Together, these results suggest that if upwelling winds intensify with climate change (Sydeman et al. 2014; Bakun et al. 2015), the increased forcing could offer a mitigating effect on future coastal depletion events (e.g. Evans et al., 2015), even as the oxygen levels in upwelled source waters decline (Doney et al. 2012; Bakun et al. 2015).
The analytical characterization of the physical-biogeochemical upwelling system presented here is possible only because of the many simplifications and idealizations in the model formulation. Many other influences on oxygen depletion, including effects of three-dimensional circulation and downwelling, and all the many complications of the biological and chemical transformations that are implicit in the model conversions between nutrients and particles, cannot be addressed within the framework of the present model, and require more sophisticated approaches with substantially more complex models. Achieving a similar level of general understanding of these more complex models remains an outstanding scientific challenge. The present work is intended as a preliminary step toward that broader understanding.

Acknowledgements  This research was supported by the National Science Foundation, Grant OCE-1130114. CSH is grateful for helpful conversations with M. Hoecker-Martinez and C. Miller. No new observational data are reported in this manuscript and all of the results and figures were obtained using standard numerical or analytical methods to solve the model equations.
Appendix

Middle-box particle concentration

The equation for the middle-box particle concentration $C_{\text{mid}}$ is most conveniently posed in terms of the scaled concentration $C'_{\text{mid}} = (S'/S)C_{\text{mid}}$:

$$\frac{dC'_{\text{mid}}}{dt} = -rC'_{\text{mid}} - \delta sC'_{\text{mid}} + \delta sC_{\text{top}} \ . \quad (A.1)$$

In this form, the sinking flux into the bottom box in (2.4) is $E_{\text{bot}} = (S'/H)C_{\text{mid}} = sC'_{\text{mid}}$, and the sinking rates and box thicknesses enter the model equations only through the single parameter $\delta$. Multiplication of (A.1a) by the integrating factor $\exp[-(r+\delta)s]t$ and integration, using $C'_{\text{mid},0} = 0$, then gives (2.6) directly. The equation (A.1a) can be convenient to retain in place of (2.6) for numerical solution. For an impulsive sinking flux from the top box $E_{\text{top}}(t) = sC_{\text{top}} = E_0\delta_D(t)$ at $t = 0$, where here $\delta_D$ is the Dirac-$\delta$ impulse, the incoming sinking flux to the bottom is $E_{\text{bot}}(t > 0) = \delta sE_0\exp[-(r+\delta)s]t$ and its total time-integral is $\int_0^\infty E_{\text{bot}}(t)dt = E_0/[1 + r/(\delta s)]$, illustrating that attenuation of the sinking flux from the top to the bottom boxes is largest for large $r$ and small $\delta s$.

Total nutrient equivalent and $N_{\text{bot}}$-$O_{\text{bot}}$ coupling

Summing the particle and nutrient equations, with the former divided by $\gamma$, gives an equation for the evolution of total nutrient equivalent, $N$, where $N = N_{\text{bot}} + N_{\text{top}} + \gamma^{-1}(C_{\text{bot}} + \delta_H^{-1}C_{\text{mid}} + C_{\text{top}})$:

$$\frac{dN}{dt} = \lambda N_s - \lambda(N_{\text{top}} + \gamma^{-1}C_{\text{top}}) - r\delta_H^{-1}C_{\text{mid}} \ . \quad (A.2)$$

Thus, in steady state, onshore nutrient transport into the bottom box is balanced by losses from offshore transport out of the top box and from middle-box respiration.
In this model, bottom nutrients and oxygen are tightly coupled: the respiration \((C_{bot})\) sink and source terms cancel from (2.1) and \(v\) times (2.3) when these equations are summed, leaving only the advective relaxation of the sum \(O_{bot} + v N_{bot}\) toward its offshore value \(O_s + v N_s\). Consequently, if, as here, bottom-box oxygen and nutrient concentrations are initially equal to their offshore values, or steady states are considered, either (2.1) or (2.3) could in principle be replaced by the relation
\[
\Delta O_{bot} = v (N_{bot} - N_s). \tag{A. 3}
\]
In general, of course, (A.3) does not hold, and to avoid possible confusion, (2.1) and (2.3) are therefore both retained throughout.

**Steady states**

The explicit (non-trivial) steady state solutions of (2.1)–(2.5) and (A.1), with \(E_{bot} = sC'_{mid}\) in (2.2), are:

\[
\Delta O_{bot} = \frac{r v}{\lambda y} C_{bot}, \tag{A.4}
\]

\[
\frac{C_{bot}}{C_{mid}} = \frac{s}{r} C_{mid}, \tag{A.5}
\]

\[
\frac{C_{mid}}{C_{top}} = \frac{\delta s}{r + \delta s} C_{top}, \tag{A.6}
\]

\[
C_{top} = \frac{(r + \delta s) \lambda}{r s + (r + \delta s) \lambda} v(N_s - N_{top}), \tag{A.7}
\]

\[
N_{top} = \frac{r + s + \lambda}{p - r - s - \lambda} K = \frac{p_s K_s}{\alpha} N_s, \tag{A.8}
\]

\[
N_{bot} = N_s + \frac{r}{\lambda y} C_{bot} = N_s + \frac{1}{v} \Delta O_{bot}. \tag{A.9}
\]
From (A.7), it follows that \( \overline{N_{top}} < N_s \) for all states with \( \overline{C_{top}} > 0 \), and from (A.4)-(A.8) that \( \overline{N_{bot}} = \overline{N_{bot}} = N_s \) and \( \overline{C_{top}} = \overline{C_{bot}} = \overline{\Delta O_{bot}} = 0 \) when \( p = p_* \), where \( p_* = (r + s + \lambda) / N_s \), because then also \( \alpha = \alpha_0 = p_* - r - s - \lambda = p_* K_s \).

Optimal sinking and constants \( P, P' \)

The sinking rate \( s = s_{opt} \) at which \( \overline{\Delta O_{bot}} \) is maximum (i.e., where \( d(\overline{\Delta O_{bot}}) / ds = 0 \), with all other parameters fixed) can be obtained as a solution of the cubic equation

\[
s^3 + as^2 + bs + c = 0, \quad (A.10)
\]

where

\[
a = 2(\lambda R - s_{max}), \quad b = s_* s_{max} - \lambda R(s_* + 3s_{max})
\]

\[
c = 2\lambda Rs_* s_{max}, \quad (A.11)
\]

with \( R = r / (r + \delta \lambda) = 1 / (1 + \delta \lambda) \). For sufficiently small \( \lambda pRK_* \) (typically satisfied because \( K \ll N_s \)), the cubic approximately factors and the approximation (5.4) for the optimal sinking rate may be obtained as a solution of the quadratic part.

The steady states with \( \overline{\Delta O_{bot}} = \overline{\Delta O_{hyp}} \) at the optimal sinking rate \( s = s_{opt} \) satisfy

\[
\overline{\Delta O_{hyp}} = \frac{\delta R s_{opt}^3}{r s_{max}(s_{opt} + \lambda R)} \nu(K + N_s), \quad (A.12)
\]

where the sinking rate \( s_{opt} \) is optimal over \( s \) for fixed \( r \) and \( \lambda \). For fixed \( p \) and \( s_{max} \), an optimal \( r \) can be found by substituting \( \lambda = p - s_{max} - r \) in (A.12) and maximizing over \( r \). This maximum of \( \overline{\Delta O_{hyp}} \), regarded as a function of \( r \), is at \( r = 0 \) for \( \delta < 1 \), and at \( r = p - s_{max} \) for \( \delta > 1 \). The resulting sufficient condition (Sec. 5.4) for the existence of hypoxic states (for some \( r \), with fixed \( p - s_{max} \) and \( s = s_{opt} \)) is

\[
p - s_{max} = r + \lambda \leq p / P, \quad (A.13)
\]
where
\[ P = \frac{\xi^2}{\xi^2 - 1} , \quad \text{(A.14)} \]

and
\[ \xi = \frac{(1 + N_s A_\delta)^{1/2} - K_s^{1/2} A_\delta}{1 - K_s A_\delta} , \quad A_\delta = \max\{1, \delta\} A \]

\[ , \quad A = \frac{v(K + N_s)}{\Delta \rho_{\text{hyp}}} . \quad \text{(A.15)} \]

Alternatively, minimization of (A.12), again with fixed \( p - s_{\text{max}} \) (i.e., \( \lambda = p - s_{\text{max}} - r \)) and \( s = s_{\text{opt}} \), gives a quartic in the variable \( q = \sqrt{p/s_{\text{max}}} \), one solution of which gives the sufficient condition for hypoxic steady states at any \( r \) or, equivalently, any \( \Lambda \) (Sec. 5.4),
\[ p - s_{\text{max}} = r + \lambda \leq p/P' . \quad \text{(A.16)} \]

The corresponding quartic equation for \( q \) is
\[ q^4 + a_q q^3 + b_q q^2 + c_q q + d_q = 0 , \quad \text{(A.17)} \]

where
\[ a_q = 2K_s^{1/2}[2K_s \delta A - (1 + \delta)] \quad , \quad \text{(A.18a)} \]
\[ b_q = 2\delta + K_s(1 - \delta)^2 - 12K_s \delta A \quad , \quad \text{(A.18b)} \]
\[ c_q = 2K_s^{1/2} \delta(3 - \delta + 6A) \quad , \quad \text{(A.18c)} \]
\[ d_q = \delta[\delta - 4(1 + A)] \quad , \quad \text{(A.18d)} \]

and \( A \) is defined in (A.15).
### Table 1. Fixed model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$N_s$</td>
<td>35 mmol m$^{-3}$</td>
<td>Upwelled source-water nutrient concentration</td>
</tr>
<tr>
<td>$C_{\text{top,0}}$</td>
<td>1 mmol m$^{-3}$</td>
<td>Top-box initial particle concentration</td>
</tr>
<tr>
<td>$N_{\text{top,0}}$</td>
<td>5 mmol m$^{-3}$</td>
<td>Top-box initial nutrient concentration</td>
</tr>
<tr>
<td>$N_{\text{bot,0}}$</td>
<td>35 mmol m$^{-3}$</td>
<td>Bottom-box initial nutrient concentration</td>
</tr>
<tr>
<td>$O_{\text{bot,0}}$</td>
<td>100 mmol m$^{-3}$</td>
<td>Bottom-box initial O$_2$ concentration</td>
</tr>
<tr>
<td>$\Delta O_{\text{hyp}}$</td>
<td>40 mmol m$^{-3}$</td>
<td>O$_2$ depletion indicating hypoxia</td>
</tr>
<tr>
<td>$\Delta O_{\text{an}}$</td>
<td>100 mmol m$^{-3}$</td>
<td>O$_2$ depletion indicating extreme hypoxia</td>
</tr>
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### Table 2. Variable model parameters with nominal or case-specific values

<table>
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<th>Parameter</th>
<th>Nominal or case-specific value</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\delta$</td>
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<td>Ratio of specific sinking rates</td>
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<tr>
<td>$r$</td>
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<td>Respiration rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.1 d$^{-1}$</td>
<td>Upwelling rate</td>
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<tr>
<td>$p$ ('Fast Particle')</td>
<td>1.1 d$^{-1}$</td>
<td>specific particle growth rate</td>
</tr>
<tr>
<td>$p$ ('Slow Particle')</td>
<td>0.45 d$^{-1}$</td>
<td>specific particle growth rate</td>
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<td>$s$ ('Fast Particle')</td>
<td>0.5 d$^{-1}$</td>
<td>specific sinking rate.</td>
</tr>
<tr>
<td>$s$ ('Slow Particle')</td>
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<td>specific sinking rate.</td>
</tr>
<tr>
<td>$K$ ('Fast Particle')</td>
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<td>limiting nutrient level</td>
</tr>
<tr>
<td>$K$ ('Slow Particle')</td>
<td>0.25 mmol m$^{-3}$</td>
<td>limiting nutrient level</td>
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</table>
References


Figure 1. Schematic of box model and total nutrient cycling. Upwelling favorable wind
stress drives offshore surface transport, parameterized by the inverse shelf-transit or
volume-renewal time $\lambda$, which is a function of the wind stress $\tau^y$, the shelf width $L$, and
the depth of the surface (and bottom) layer $H$. Upwelling brings nutrients to the surface,
driving particle production. Some of the upwelled nutrient flux is lost offshore as
nutrients ($\lambda N_{\text{top}}$) or particles ($\lambda C_{\text{top}}$), some exported to the bottom as sinking particle flux
($E_{\text{bot}}$), and some remineralized in the mid-watercolumn ($r C_{\text{mid}}$). Offshore surface
transport is compensated by onshore transport in the bottom box, bringing source
nutrients ($N_s$) and oxygen ($O_s$) into the system.
Figure 2. Temporal evolution of model with constant forcing at $\bar{\alpha} = 0.1 \text{ d}^{-1}$ with other parameters fixed at the FP parameter-set values. a) Evolution of the deep-box oxygen depletion and particle accumulation; and b) evolution of the surface particle accumulation and surface and deep nutrient concentrations. Other curves are shown for comparison: the deep-box oxygen depletion predicted from the exponential approximation (light red line, Figure 2a); the oxygen depletion for the SP parameter set (light black line, same as heavy read line in Figure 3a); and the $C_{\text{top}}$ predicted from the exponential approximation (Fig. 2b, light dashed green line). Large solid circles at day 150 represent the $\Delta O_{\text{bot}}$, $C_{\text{bot}}$, and $N_{\text{bot}}$ steady state solutions.
Figure 3. Temporal evolution of model with constant forcing at $\square = 0.1$ d$^{-1}$ as in Figure 2, for the SP parameter set. a) Evolution of the deep-box oxygen depletion and particle accumulation; and b) evolution of the surface particle accumulation and surface and deep nutrient concentrations. Other curves are shown for comparison: the deep-box oxygen depletion predicted from the exponential approximation (light red line, Figure 3a); the $C_{top}$ predicted from the exponential approximation (Fig. 2b, light dashed green line); and the $C_{top}$ for the FP parameter set (Fig. 3b, light dashed black line; same as heavy dashed green line in Fig. 3b).
Figure 4. a) Efficiency of retention \([c(t)\text{ from eq. (3.2a)}]\) of respiratory potential for the FP (red) and SP (blue) cases shown above, and b) Fractional export of respiratory potential via \(N\) (dashed lines) and \(C\) (solid lines) export from the top box.
Figure 5. Steady-state oxygen depletion with other parameters as for the FP (solid line) and SP (dashed) parameter sets vs. variable (a) production rate $p$, (b) upwelling rate $\lambda$ and (c) sinking rate $s$. Black circles show solutions corresponding to the model results from Figs 2 and 3, with the fixed FP and SP values of $p$, $\lambda$ and $s$. 
Figure 6. Timescales for oxygen depletion vs. (a) \( \lambda_0 \) and (b) \( s \) for other parameters fixed at their FP-case values. Light and heavy solid lines show the times for the full-model to reach the \( \Delta O_{\text{bot}} = 100 \) and 40 mmol m\(^{-3}\), respectively, while the corresponding dashed lines show the same timescales for the exponential approximations. The light dotted line shows the dependence of the surface exponential-growth term, \( \alpha \).
Figure 7. Combined contour plot of steady-state depletion and timescale metrics vs. dimensional productivity and sinking rates, $p$ and $s$ (d$^{-1}$), for a hypoxic threshold value $\Delta O_{hyp} = 100$ mmol m$^{-3}$ and the (a) FP and (b) SP values of all other parameters. Steady state bottom-oxygen depletion $\Delta O_{bot}$ (mmol m$^{-3}$; labeled line contours) is shown where it is less than $\Delta O_{hyp}$. Estimated time to hypoxia $t_{hyp}$ (d; color-filled contours) is shown where the steady-state depletion is greater than $\Delta O_{hyp}$. Other parameter values are: $(r, \lambda) = (0.2, 0.1)$ d$^{-1}$, $\delta = 1/3$, $N_s = 35$ mmol m$^{-3}$ and (a) $K = 1$, (b) $K = 0.25$ mmol m$^{-3}$. The FP and SP values of $(p, s)$ are indicated (black dots). The critical lines $s = s_* = pN_s - r - \lambda$ are also shown (solid); for $s > s_*$, there is no growth or retention ($C_{top} = C_{bot} = \Delta O_{bot} = 0$).
Figure 8. Combined contour plots as in Fig. 7, but vs. dimensionless, relative maximal accumulation and sinking rates, $\tilde{\alpha}$ and $\tilde{s}$. Steady state bottom-oxygen depletion $\Delta O_{\text{bot}}$ (mmol m$^{-3}$; labeled line contours) is shown where it is less than $\Delta O_{\text{hyp}} = 100$ mmol m$^{-3}$. Estimated dimensionless time to hypoxia $\tilde{t}_{\text{hyp}} = (r + \lambda)t_{\text{hyp}}$ (d; color-filled contours) is shown where the steady-state depletion is greater than $\Delta O_{\text{hyp}}$. Other parameters are: $N_s = 35$ mmol m$^{-3}$, $(\delta \Lambda) = (1/3,0.5)$ and (a) $K^* = 0.028$ ($K = 1$ mmol m$^{-3}$), (b) $K^* = 0.007$ ($K = 0.25$ mmol m$^{-3}$). The FP and SP values of $(\tilde{\alpha}, \tilde{s})$ are indicated (black dots). The critical lines $\tilde{s} = s^*$ (solid) and $\tilde{s} = s_{\text{opt}}$ (dashed; blue: $\tilde{\alpha} < P_{\tilde{\alpha}}$, green: $P_{\tilde{\alpha}} < \tilde{\alpha} < P'_{\tilde{\alpha}}$, red: $\tilde{\alpha} > P'_{\tilde{\alpha}}$) are also shown. The dimensionless time $\tilde{t} = 45$ corresponds to 150 d for the FP and SP value of $r + \lambda = 0.3$ d$^{-1}$. 
Figure 9. Optimal (a) dimensionless sinking rate $\tilde{s}_{opt}$ and (b) dimensionless maximal accumulation rate $\tilde{\alpha}_{opt} = \tilde{p} - 1 - \tilde{s}_{opt}$ vs. $(\tilde{p}, \tilde{K})$. In both, the FP (black dot) and SP (white dot) values of $(\tilde{p}, \tilde{K})$ are indicated. The hypoxia criteria $\tilde{p} = P$ (green solid line) and $\tilde{p} = P'$ (red solid line) are shown and the corresponding regimes are indicated (white, green, red labels) for $\delta = 1/3$. In (b), the criteria are also shown for $\delta = 5/3$ (dotted green, dotted red).
Figure 10. Steady-state oxygen drawdown $\Delta O_{\text{bot}}$ (mmol m$^{-3}$) at the optimal sinking rate $\tilde{s}_{\text{opt}}$ vs. ($\delta$, log$_{10} \Lambda$), with $p$ and $K$ as for the (a) FP and (b) SP examples. The 100 mmol m$^{-3}$ contour (white) and the (a) FP and (b) SP values (white dot) of ($\delta$, log$_{10} \Lambda$) are indicated.
Figure 11. Exponential drawdown timescale $\tilde{\tau}_{\text{hyp}}$ at the optimal sinking rate $\tilde{s}_{\text{opt}}$ vs. $(\delta, \log_{10} \Lambda)$, with $\rho$ and $K$ as for the (a) FP and (b) SP examples. The (a) FP and (b) SP values of $(\delta, \log_{10} \Lambda)$ are indicated (white dots). For the FP and SP value $r + \lambda = 0.3$ d$^{-1}$, a nominal upwelling season length of 150 d corresponds to a dimensionless time $\tilde{\tau} = 45$. 
Figure 12. Steady-state hypoxia-criteria constants (a) $P$ and (b) $P'$ vs. $(\delta,K^*)$. The FP and SP values of $(\delta,K^*)$ are indicated (black dots).
Figure 13. Solutions for FP as in Fig. 2 but for oscillatory forcing as in (6.1). a) Time-dependent solutions for $\Delta O_{bot}$ and $C_{bot}$ (red and green solid lines, respectively). b) Time-dependent solutions for $C_{top}$ and $N_{top}$ (green and blue dashed lines) and $N_{bot}$ (blue solid line). Thin black lines show the time-dependent solutions for $\Delta O_{bot}$ (Fig. 13a, solid line) and $C_{top}$ (14b, dashed line) in the constant-forcing case (from Figures 2a,b).
Figure 14. a) Retention $c(t)$ from Eq. (3.2) and b) export fractions for the variable forcing scenario with the FP parameter set. Red lines are the same as in Figure 4 for the FP case and are shown for comparison.